REAL NUMBERS 1

Euclid's Division Algorithm

IMPORTANT TERMS AND CONCEPTS

Euclid's division algorithm is a technique to compute the highest common factor (HCF) of two given positive integers.

It is used to obtain the HCF of two positive integers say m and n, where $m > n$.

Step-I: We find the whole numbers q are r such that

$$m = nq + r, 0 \leq r < n$$

Step-II: If $r = 0$, then n is the HCF of m and n.

Step III: If $r \neq 0$, then apply division lemma to n and r and obtain two whole numbers p and r_1 such that

$$n = r \times p + r_1$$

Step-IV: If $r_1 = 0$ then r is the HCF of m and n.

Step-V: If $r_1 \neq 0$, then apply the division lemma to p and r_1 and continue the above process till the remainder is zero. The division at that stage is the HCF of m and n.

Fundamental Theorem of Arithmetic

IMPORTANT TERMS AND CONCEPTS

A composite number can be expressed as a product of primes and this decomposition is unique, apart from the order in which prime factors occurs.

For two positive numbers a and b

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b.$$

Irrational Numbers and Nature of Decimal Expansions of Rational Numbers

IMPORTANT TERMS AND CONCEPTS

(i) A number which can not be expressed in the form $\frac{m}{n}$ where m and n are integers, $q \neq 0$ is called irrational number.

(ii) Decimal expansion of a rational number is either terminating or non terminating but repeating.

Zeroes of a Polynomial

IMPORTANT TERMS AND CONCEPTS

k is said to be zero of a polynomial $p(x)$ if $p(k) = 0$

Geometrical meaning of the zeroes of polynomial

x coordinate of the point of intersection of the graph of a polynomial with x-axis is known as zero of the polynomial.

Number of zeroes of a polynomials is equal to number of points of intersection of the graph of polynomial with x-axis.

Relationship Between the Zeroes and the Coefficient of a Quadratic Polynomial

IMPORTANT TERMS AND CONCEPTS

If α and β are zeores of a quadratic polynomial $p(x) = ax^2 + bx + c$

Then

(i) $\alpha + \beta = \frac{-b}{a}$ $\Rightarrow$ sum of zeroes $= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

(ii) $\alpha.\beta = \frac{c}{a}$ $\Rightarrow$ Product of zeroes $= \frac{\text{constant term}}{\text{coefficent of } x^2}$

If α, β are zeroes of a quadratic polynomial then

$$p(x) = k[x^2 - (\alpha + \beta)x + \alpha\beta]$$

$$\Rightarrow \quad p(x) = k[x^2 - (\text{sum of zeroes})x + \text{product of zeroes}]$$

Where k is any non-zero real number. [Generally we take $k = 1$]

If α, β and γ are zeroes of a cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$

Then

(i) $\alpha + \beta + \gamma = \frac{-b}{a}$ $\Rightarrow$ sum of zeroes $= \frac{-\text{coefficient of } x^2}{\text{coefficient of } x^3}$

(ii) $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$ $\Rightarrow$ sum of the product of zeroes taken two at a time $= \frac{\text{coefficient of } x}{\text{coefficient of } x^3}$

(iii) $\alpha\beta\gamma = \frac{-d}{a}$ $\Rightarrow$ Product of zeroes $= \frac{-\text{constant term}}{\text{coefficient of } x^3}$

$$\boxed{\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \; ; \; \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}$$

If α, β and γ are the zeroes of a cubic polynomial $p(x)$ then

$$p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \alpha\gamma + \beta\gamma)x - (\alpha\beta\gamma)$$

Division Algorithm for Polynomials

IMPORTANT TERMS AND CONCEPTS

If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$$p(x) = g(x).\, q(x) + r(x)$$

(i) If $r(x) = 0$, then $g(x)$ is a factor of $p(x)$

(ii) If $r(x) > 0$, then degree of $r(x) <$ degree of $g(x)$

(iii) Degree of $g(x)$ + degree of $q(x)$ = degree of $p(x)$

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

3

Graphical Method of Solving Pair of Linear Equations in Two Variables

IMPORTANT TERMS AND CONCEPTS

General form of pair of linear equation in two variables is

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

METHOD OF SOLVING PAIR OF LINEAR EQUATIONS IN TWO VARIABLES:

Graphical method:

Step I: Get three solutions of each of given linear equation in two variables.

Step II: Plot these points on graph in order to draw the lines representing these equations.

(*i*) If graph represent two intersecting lines then point of intersection is the solution of the pair of linear equations.

(*ii*) If graph represent two parallel lines then pair of linear equations has no common solution.

(*iii*) If graph represents two coincident lines then pair of linear equations has infinitely many solutions.

Note: For a pair of linear equation in two variables: $a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$

(*i*) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then the two lines will intersect at one point. Such a pair of linear equation has a unique solution and is known as consistent pair of linear equations.

(*ii*) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then the graph will represent two parallel lines. Such a pair of linear equations in two variables has no common solution and is known as inconsistent pair of linear equations.

(*iii*) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then the graph will represent two coinciding lines. Such a pair of linear equation in two variables has infinitely many solutions and is known as dependent and consistent pair of linear equation in two variables.

Algebraic Method of Solving Pair of Linear Equations in Two Variables

IMPORTANT TERMS AND CONCEPTS

Substitution method

In this method we use the following steps.

(*i*) Find the value of one variable in terms of other variable from one of the equation.

(*ii*) Substitute this value of the variable in other equation. It reduces to equation in one variable. We can easily solve to find the value of that variable.

(*a*) If after the IInd step we get a true equality which is independent of the variable then pair of linear equation has infinite many solution.

(*b*) If after the IInd step we get a false equality which is independent of variable, then pair of linear equation has no common solution.

Method of Elimination

IMPORTANT TERMS AND CONCEPTS

Steps used in this method are:

(*i*) Make the coefficients of one of the variable numerically equal by multiplying both the equations by a suitable non-zero real number.

(*ii*) Add or subtract both the equations so that one of the variable is eliminated and only one variable is left.

(*iii*) Solve the equation in one variable so obtained to get the value of one variable.

(*iv*) Substitute the value of the variable in any of the given equations to get the value of other variable.

> **Note:** (*a*) If we get a true equality without any variable after the IInd step, then pair of linear equations has infinite many solution.
>
> (*b*) If we get a false equality without any variable after the IInd step, then pair of linear equations has no common solution.

Cross Multiplication Method

IMPORTANT TERMS AND CONCEPTS

For a pair of linear equations

i.e. $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then following steps are used to find the values of x and y.

Step I: Make the diagram as follows:

$$\begin{array}{ccccccc} & x & & y & & 1 & \\ b_1 & \times & c_1 & \times & a_1 & \times & b_1 \\ b_2 & & c_2 & & a_2 & & b_2 \end{array}$$

Step II: Write the equation

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Step III: Obtain the value of x and y as follows:

(*i*) $\frac{x}{b_1c_2 - b_2c_1} = \frac{1}{a_1b_2 - a_2b_1} \Rightarrow x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$

(*ii*) $\frac{y}{c_1a_2 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1} \Rightarrow y = \frac{c_1a_2 - a_1c_2}{a_1b_2 - a_2b_1}$

> **Note:** For the pair of linear equation of the type $a_1x + b_1y + c_1 = 0$, and $a_2x + b_2y + c_2 = 0$, where $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, by cross multiplication method we have $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$.

Equation Reducible to Pair of a Linear Equations

IMPORTANT TERMS AND CONCEPTS

If the given pair of equations is not linear, but can be reduced to linear form by making some suitable substitutions, then we reduce the pair of equations to linear form and solve it.

For example: $\frac{2}{x} + \frac{3}{y} = 5$ and $\frac{4}{x} + \frac{5}{y} = 9$

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$, then the above equations become $2a + 3b = 5$ and $4a + 5b = 9$.

Solution of a Quadratic Equation by Factorisation

To find the solution of a quadratic equation by factorisation, we first write the given quadratic equation as product of two linear factors by splitting the middle term. By equating each factor to zero we get possible solutions/roots of the given quadratic equation.

For Example: Find the roots of the equation $x^2 - 5x + 6 = 0$ by factorisation.

Solution: Let us Ist split the middle term

$$x^2 - 3x - 2x + 6 = 0$$

$$x(x-3) - 2(x-3) = 0 \Rightarrow (x-3)(x-2) = 0$$

Put $(x-3) = 0 \Rightarrow x = 3$ and $x - 2 = 0 \Rightarrow x = 2$

So, $x = 2$ and $x = 3$ are solution of equation.

4 QUADRATIC EQUATIONS

Introduction to Quadratic Polynomials and Quadratic Equations

Quadratic polynomial: A polynomial of degree two is known as quadratic polynomial. $p(x) = ax^2 + bx + c$ is a quadratic polynomial where $a \neq 0$ and a, b, c are real numbers.

Quadratic equation: If $p(x)$ is a quadratic polynomial then $p(x) = 0$ is known as a quadratic equation, i.e. $ax^2 + bx + c = 0$ as known as a quadratic equation. Here $a \neq 0$ and a, b, c are real numbers. e.g. $2x^2 - 7x + 5 = 0$ is quadratic equation.

Here a = coefficient of $x^2 = 2$; b = coefficient of $x = -7$

c = term independent of $x = 5$

Solving a Quadratic Equation by Completing the Square

Let quadratic equation be $ax^2 + bx + c = 0$

Step 1. Make the coefficient of x^2 equal to 1, i.e. $a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Step 2. Add and subtract the square of half of coefficient of x,

i.e. $\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a}\right) = 0$

Step 3. Express in the form of a perfect square added with a constant,

i.e. $\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}\right] = 0$

Step 4. $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$

Step 5. $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$ **Step 6.** $x = \frac{-b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \Rightarrow x = \frac{-b \pm \sqrt{b^2 - ac}}{2a}$

Note: If $\frac{b^2}{4a^2} - \frac{c}{a}$ is negative then equation has no real roots.

Solution of Quadratic Equation using Quadratic Formula or Discriminant Method

Let quadratic equation be $ax^2 + bx + c = 0$

Step 1. Find $D = b^2 - 4ac$.

Step 2. (*i*) If $D > 0$, Solution/roots of the quadratic equation are given by

$$x = \frac{-b + \sqrt{D}}{2a}, \frac{-b - \sqrt{D}}{2a}$$

(*ii*) If $D = 0$, Solution/roots of the quadratic equation are given by $x = \frac{-b}{2a}$.

(*iii*) If $D < 0$, equation has no real roots.

Nature of the Roots of Quadratic Equation and Application of Quadratic Equations

- Let given quadratic equations is $ax^2 + bx + c = 0$. $a \neq 0$

  Here $D = b^2 - 4ac$

  (*i*) If $D > 0$, then quadratic equation has two unequal real roots.

  (*ii*) If $D = 0$, then quadratic equation has two equal real roots.

  (*iii*) If $D < 0$, then quadratic equation has no real roots.

- If α, β are roots of the quadratic equation $ax^2 + bx + c = 0, a \neq 0$, then

  (*i*) Sum of roots $= \alpha + \beta = \frac{-b}{a}$

  (*ii*) Product of roots $= \alpha \cdot \beta = \frac{c}{a}$

  (*iii*) $ax^2 + bx + c = (x - \alpha)(x - \beta)$

- If α, β are roots of a quadratic equation then quadratic equation is given by x^2 – (sum of the roots)x + product of the roots $= 0$

  $\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$

- Let given quadratic equation is $ax^2 + bx + c = 0, a \neq 0$

  (*i*) If a, b and c are rational and D, i.e. $b^2 - 4ac$ is a perfect square then roots of the quadratic equation are rational.

  (*ii*) If a, b and c are rational and D, i.e. $b^2 - 4ac > 0$ and D is not a perfect square then both the roots of the quadratic equation are conjugate irrational (irrational numbers of the type $2 + \sqrt{5}$ and $2 - \sqrt{5}$ are known as conjugate irrational numbers).

  (*iii*) If $a + b + c = 0$ then roots of quadratic equation are 1 and $\frac{c}{a}$.

  (*iv*) If $a + c = b$ or $a - b + c = 0$ then roots of the quadratic equation are -1 and $-\frac{c}{a}$.